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SUBTRACTION MADE EASY FOR PUPILS WITH LEARNING DIFFICULTIES

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Abstract

Subtraction is the foundational concept in Mathematics. Unfortunately, pupils with learning difficulties often find it challenging as it entails counting backward. Going gets tough for these pupils when the number gets bigger and involves regrouping. These phenomena were observed among five pupils in a second grade remedial class in a suburban primary school. From the error analysis and the subsequent individual interviews, these pupils showed a lack of understanding of the place value and subtraction concept although they generally grasped the concept of one to one correspondence. In addition they were found to have retention difficulty and thus lost interest and patience in learning mathematical skills. Therefore, this action research was conducted to improve the subtraction skills for these five pupils as research participants. The focused skill was subtraction with regrouping within 100. For this purpose, a unique subtraction board was devised based on Bruner's ConcretePictorial-Abstract (CPA) concept. In the first phase, participants revisited the concept of place value using a story. Next, tangible objects were used for concrete concept formation followed by drawing and finally abstract numbers in a progressive manner depending on participants' pace and progress. A catchy song was created to enhance skill retention. Besides, drills and practice were made more fun in the form of computer games devised by the authors. The intervention was conducted 5 days a week, 1 hour per day for 2 weeks. Pretest and posttest results showed significant improvement after the intervention. Findings from ensuing interviews and observations demonstrated that repeated use of manipulatives helped participants visualize subtraction of objects and gave them clearer picture on place value. The findings indicated that scaffolds of manipulatives and drawings which were slowly removed could help participants in understanding abstract mathematics concept while the song and computer games served as memory enhancer and intriguing drills.

Keywords: subtraction, concrete-pictorial-abstract approach, remedial pupils

Introduction

Basic numeracy skills are important life skills to be acquired by pupils to function fully and live a rewarding life in this modern society. In line with this concern, *Literacy and Numeracy Screening* (LINUS) programme was initiated in 2010 by Malaysian Ministry

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of Education. LINUS is a commendable effort to ensure all pupils acquire basic literacy and numeracy skills after three years of primary education. This is essential as some pupils face difficulties in developing and acquiring these skills at this early stage of education.

Mathematical difficulties need to be tackled early so as to prevent the development of negative attitudes and mathematics anxiety. Therefore, teachers need to equip themselves with content knowledge and developmentally appropriate strategies to support these pupils' learning. Continuing professional development through action research is an ongoing process for teachers to reflect and expand their knowledge base and build their pedagogical skills. The process itself is beneficial for both the teachers as well as the pupils. Therefore this paper reports an action research to enhance remedial pupils' subtraction skill with specially designed intervention.

Reflection on Teaching and Learning

From the experience in teaching Mathematics to remedial pupils, it is found that these pupils commonly face the problems of inconsistency in computational skill, inadequacy in understanding number facts, inability to remember basic mathematic facts or procedures and are very slow to retrieve facts or procedures which they have learnt. In addition, they are easily distracted during Mathematics lesson or while doing Mathematics tasks. They appear to be easily bored and are reluctant to answer a worksheet full of exercises.

This was apparent when teaching these pupils subtraction. They struggled especially with two-digit subtraction with regrouping. They failed to grasp the idea of "*borrowing*" although the concept had been retaught for many times using different examples. When Dienes Blocks were used as manipulatives, some pupils appeared to have understood the concept of "*borrowing*". However, they could not apply the concept into their computational skill. After several attempts, these pupils began to show signs of anxiety and frustration.

Typical errors identified in their worksheets were inversion errors (taking the larger number to subtract the smaller number regardless of their positions); basic number facts errors (adding instead of subtracting); insufficient knowledge about place values and wrong calculation due to carelessness (Figure 1). According to Beckett, McIntosh, Byrd & McKinney (2011), errors patterns are misconceptions and erroneous understandings pupils make when learning new mathematical concepts. This can be due to wrong learning transfer of mathematical concept or fail to master the concept of place values in base-ten number system.

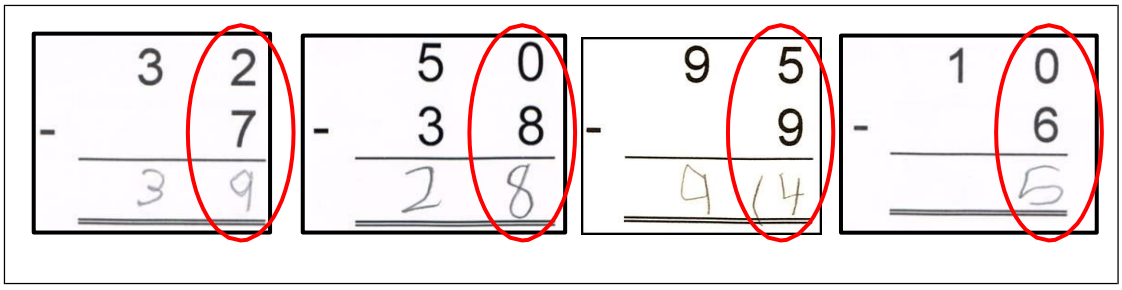

Figure 1 – Examples of pupils' errors in subtraction with regrouping

Previous studies show that successful pupils often hit early on a strategy for remembering simple mathematic facts, whereas less successful pupils lack such skills. (Carnine & Stein, 1981). These unsuccessful learners need guidance to associate facts and rules in logical order to help memory retention (Geary, 2003; Gersten, Jordan & Flojo, 2005; Steinberg, 1985; Thorton & Smith, 1988; Van Houten, 1993).

Voza (2011) suggests that to teach subtraction with regrouping, teachers should start with number sense and focus on what subtraction means. Teachers could use manipulatives and physical representations of numbers before moving on to numerical problems number lines. To build memory retention, the subtraction concept has to be taught in context (Janes & Strong, 2014).

Literature in mathematics instruction shows that the optimal presentation sequence to teach new mathematical content is through the concrete-pictorial-abstract (CPA) approach (Miller & Mercer, 1993) originated from the work of Jerome Bruner (Sousa (2008). Gujarati (2013) supported this approach and claimed that using this approach, pupils are experiencing and discovering mathematics rather than simply regurgitating it. A study by Flores (2009) applied concrete-representational-abstract (CRA) instructional sequence on third grade pupils with learning disabilities. The finding indicated that pupils could demonstrate their understanding of the regrouping procedures instead of memorizing steps.

Automaticity or direct retrieval of facts is essential as this will reduce working memory load during mathematical processes (Dehn, 2008). Fluency in computation is important because humans have a limited information-processing capacity. Fluent retrieval of mathematic facts must be developed to enable the development of higher-order mathematical skills (Resnick, 1983). Once procedures are automatized, they require little conscious effort to use, which, in turn, frees attentional and working memory resources for other higher order problem solving (Geary, 1995).

In brief, effective intervention should focus on contextual learning, applying a three step progressive CPA sequence and interesting rehearsals to achieve automaticity or fluency in computation.

Research Focus

The intervention design aimed to directly tackle participants' problems. Therefore the research focused on participants' inadequacy understanding about subtraction with regrouping, misconceptions about subtraction facts and place values, inefficiency in computation or procedural knowledge, and lack of motivation in learning. As these problems were intertwined and related, intervention would include a variety of strategies to provide context to relate, give hands-on and visual representations to demonstrate facts, use songs to remind computation procedures, incorporate computer games for fluency and automaticity. The whole package of intervention would apply brain-compatible learning strategies (Caine, Caine, McClintic & Klimek, 2009) to engage participants in learning.

Research Questions

The purpose of the intervention is to enhance the understanding, skills and motivation of remedial pupils in subtraction with regrouping. Therefore this action research sought to answer these research questions.

1. Is the intervention effective in enhancing pupils' skills of subtraction with regrouping?
2. Is the intervention effective in enhancing pupils' understanding about the concept of subtraction with regrouping?
3. Is the intervention effective in instilling pupils' interest in learning subtraction with regrouping?

Methodology

This study applied an action research design which involves a cycle of reflecting on problems, planning for intervention, implementing the intervention, observing the effects of intervention and evaluation of the intervention.

Research Participants

Participants of the action research involved five Year 2 pupils in a Chinese national-typed primary school in Selangor. These participants were pupils who passed the numeracy screening test in the beginning of the year but had mathematical difficulties in mainstream classrooms. They often failed in mathematic tests and were generally weak in all other academic subjects.

Table 1 – Participants' profile

Participants	Age	Gender	Ethnic	Characteristic
A	8	Male	Chinese	Leader among this five participants Very active Poor concentration
B	8	Male	Chinese	Faithful follower of Participant A Quiet Slow compared to other four participants
C	8	Male	Chinese	Poor physical coordination Requires more time in writing and counting compared to other four participants
D	8	Female	Japanese-Chinese	Active in learning Poor memory Friendly and helpful
E	8	Female	Japanese-Chinese	Loves to tattle about others Poor concentration • Wanders off from desk during lesson

Intervention Procedures

The intervention lasted for 2 weeks, 5 days per week and 1 hour per day. There were a few stages of the intervention. First stage provided a contextual situation for a revisit on place values and subtraction concept. Second stage involved participants in stepby-step hands-on activities using subtraction kit from concrete to pictorial and abstract phases.

Stage three involved a "rap and sing" session whereby participants learnt a rap song specially designed as a reminder of the subtraction procedures. Fourth stage reinforced the skill learnt with computer game challenge.

Stage 1 Fun storytelling – Contextual learning for concept introduction

A fun learning story was introduced to pupil. The story aimed to enhance pupil's understanding on place value of ones and tens as well as subtraction with regrouping (Table 2). To make the story more attractive, it was illustrated with pictures and animation using *PowerPoint* presentation (Figure 2).

Table 2 – Contextual story and mathematic concept

Story	Mathematic Concept
Chubby Bobby was lost in the jungle. He found 2 special banana trees in a row. These trees belonged to Swingy Monkey. The trees had special labels on them, "ones" on the tree on the right and "tens" on the tree on theOn the "ones"left. tree, there were 9 bananas on it. On the "tens" tree, there were 9 bunches of bananas. Each bunch had 10 bananas. That's why it is called the "tens" tree! So there were all together 99 bananas.	**90 + 9 = 99**
Chubby Bobby was amazed by the trees and their special names. But poor Chubby Bobby was very hungry now. He saw the Swingy Monkey busy plucking the bananas from the "ones" tree to give to his 8 baby monkeys. Chubby Bobby asks Swingy Monkey, "May I have 2 bananas, please?" Uh oh... but there is only one left on the "ones" tree. "1 not enough to give 2." "Well, we need to borrow from the "tens" tree."	**9 – 8 = 1**
"Let's take one bunch from the "tens" tree to bring to the "ones" tree. Now we have 10 bananas, plus the 1 banana left, there are 11 bananas on the "ones" tree. You can take 2 bananas now, Chubby Bobby." "Thanks a lot."	**10 + 1 = 11**
"Well, how many bananas are left on the "ones" tree?" "11 take away 2, we now have 9!"	**11 – 2 = 9**
"How about the "tens" tree?"	**90 – 10 = 80**
"There were 9 bunches. Given one bunch to "ones" tree, now there is 8 bunches left".	
"So after giving 8 bananas to baby monkeys, there were 91; and after giving 2 to Chubby Bobby, there were 89 left.	**99 – 8 = 91** **91 – 2 = 89**

Figure 2 – Examples of screens taken from animated illustration of the story

Stage 2: Concrete-Pictorial-Abstract (CPA)

A subtraction kit was designed based on Bruner's Concrete-Pictorial-Abstract (CPA) concept. Participants were introduced to the subtraction board. They would be told that this was like the Swingy Monkey's banana trees with "tens" on the left column and "ones" on the right column. On top of the "tens" column, there was a box with ice- cream sticks tied up in bundles of ten. On the "ones" column, there was another box with ice-cream sticks, not tied up.

There were 3 phases in this stage First, participants were asked to do subtraction exercises using ice-cream sticks as concrete manipulatives (Figure 3). The exercises would start from an easy set (2 digits minus 1 digit within 50) to a moderate set (2 digits minus 1 digit within 100) and a more difficult set (2 digits minus 2 digits within 50) and a challenging set (2 digits minus 2 digits within 100).

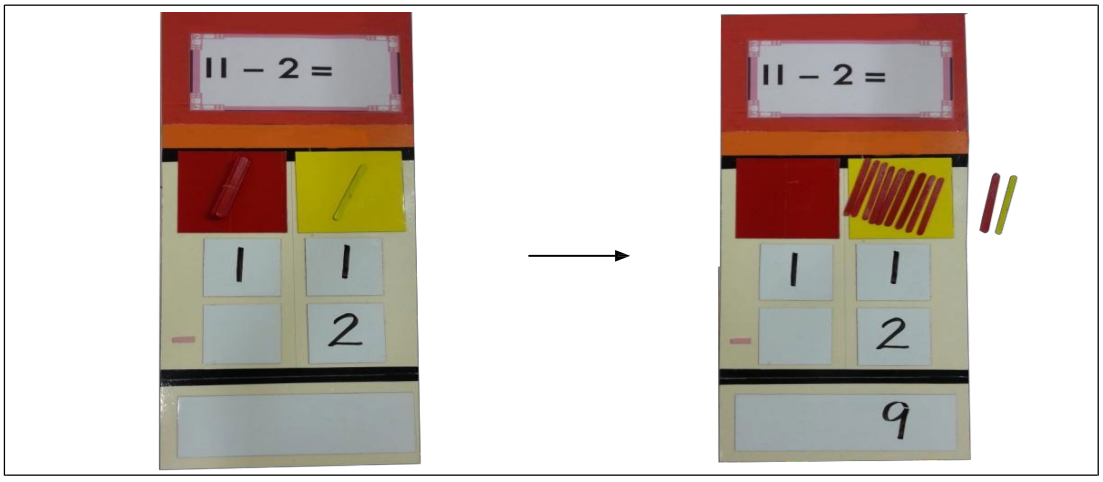

Figure 3 – Examples of board usage in concrete phase

After participants achieved 90% accuracy or better, instead of using ice-cream sticks, they were asked to represent the sticks by drawing "/" to help in counting.

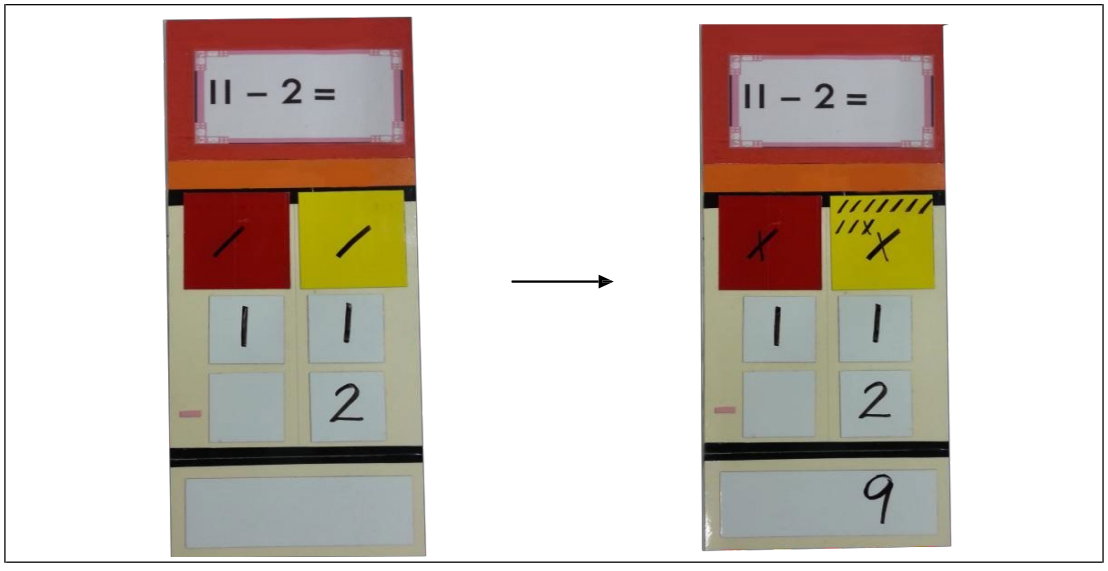

Figure 4 – Examples of board usage in pictorial phase

The participants then moved on to the abstract phase when they were ready to count from memory.

Stage 3: Sing it and Rap it!

A song was designed as a reminder to participants about the steps of doing subtraction with regrouping and to correct their earlier misconceptions. It was a simple song, which could be rapped or sung to the tune of a popular Chinese children song "三轮车" (Figure 4).

借位减法要牢记
(Remember this, in subtraction) 先从个位来减起
(Start with ones)
上面数字比较大
(More on top)
那就可以直接减
(minus it straight)
上面数字比较小
(Less on top)
那就得向十位借
(Borrow from tens)
上下数字都一样
(Same numbers)
答案就是零鸡蛋
(No more left)

Figure 5 – Lyrics of subtraction rap song

Stage 4: Subtraction Challenge for Automaticity and Speed

To boost participants' interest in doing more drills and practice so as to perfect their skills and to encourage automaticity in fact retrieval, a computer game was devised. There were 5 stations in the games arranged from easy to difficult levels. Each station has 20 questions to be solved within a time limit. The time limit could be adjusted to give more challenge to the participants but within their capabilities.

Figure 6 – Computer games devised by researcher to be use in the last phase

Data Collection

30-item subtraction tests were administered before and after intervention to evaluate the effectiveness of the intervention on participants' subtraction with regrouping skills. The 30-item subtraction test comprises 5 items 2 digits minus 1 or 2 digits questions without regrouping, 5 items 2 digits minus 1 digit questions with regrouping within 50, 5 items 2 digits minus 1 digit questions with regrouping within 100, 5 items 2 digits minus 2 digits questions with regrouping within 50 and 10 items 2 digits minus 2 digits questions with regrouping within 100.

Semi-structured interviews were conducted before and after intervention to gain insight about participants' understanding of the subtraction concepts through their explanation of how they work on the question. Further questions were asked to probe how or why participants carried out the procedures.

Unstructured direct observations were applied before and during the intervention to evaluate participants' motivation and engagement in learning. Participants' classroom behaviours were observed in naturalistic setting and recorded in the form of field notes.

Research Findings

Subtraction Skill Test

Subtraction Skill Test administered before and after intervention was shown in Figure 6. All the participants showed drastic improvement after the intervention, scoring from 93% to 100%. The increase in percentage ranged from 69.7% (Participant A) to 83.4% (Participant D).

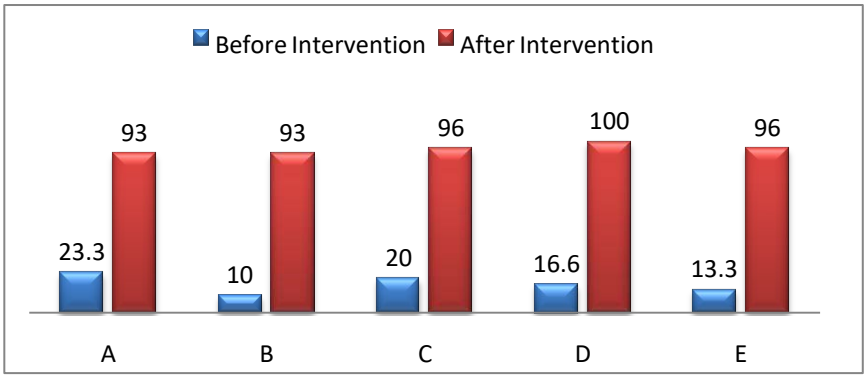

Figure 7 – Comparison of subtraction skill test scores before and after intervention

Findings from Semi-structured Interviews

Interviews were conducted individually to gain insight on participants' understanding of the concept about subtraction with regrouping by asking them how they derive the answer for the question given ("34 – 9 = ?"). They had to think aloud and explain the procedures of getting the answer.

Data collected demonstrated that some participants had misconceptions about the computation procedures. For example: "*I take 9 and minus 4, so my answer is 34*" (participant A) or "*9 minus 4 equals 5, 35*" (participant C). When further question was asked to probe why they minus 4 from 9, participant A answered, "*Because 9 is bigger*" while participant C replied, "*because 4 cannot minus 9*". This indicated a

misconception and negative learning transfer of "*bigger number should minus smaller number*". It also indicated inadequate understanding of subtraction concept.

Participant D showed some understanding about "borrowing" concept of regrouping but demonstrated incomplete knowledge and skill. "*4 cannot minus 9. So we need to borrow from tens... then 10 minus 9 is 1, so the answer is 21."*

On the other hand, participant E was confused about the place value and subtraction concept as the answer given was 313. *"4 plus 9 is 13. And this is 3, so 313."* When asked about why she plus instead of subtract, she realised that she misinterpreted the symbol. However, she was unable to do self-correction for the answer. Participant B did not provide any answer. "*I don't know how to do this*." This clearly showed that he could not attempt the question and had not mastered enough skills to do it.

Similar interviews were conducted after two-week intervention. The participants showed great improvement in the understanding as compared to before intervention. All of them got the answers correct. The explanation given showed increased understanding of the procedures.

"*We need to borrow from tens. Then only minus 9. (counting...) Answer is 25*." (Participant A)

"Borrow from here (pointing at 3) and put ten into here (pointing at 4) then minus. (painstakingly...using pictorial representation) The answer is 25." (Participant B)

"4 cannot minus 9 because 9 is more. So we borrow from tens. And now 4 become 14. 14 minus 9. We get... 5. And 3 minus 1 is 2." (Participant C) *"(Rapping the song himself) This one cannot minus because less on top. We borrow from tens! So after we borrow from 30, 30 will become 20 and here (pointing at ones) 10 plus 4 will become 14 then minus... it is 5. So... 25 "* (Participant D) "*Now Bobby wants to eat 9 bananas but the tree has only 4 bananas. How? We must borrow from the tree beside it. 3 becomes 2 and 4 becomes 14. Give Bobby 9 bananas and now we still have 25 bananas! Haha..."* (Participant E)

It was amazing to see the participants relating their experience in the intervention to be applied in their working to get the answer. This demonstrated a positive transfer of knowledge.

Findings on Direct Observations

Before the intervention, participants were generally demotivated. They were easily distracted during the remedial lessons. In addition, they were impatient to listen to explanation. They disliked drills and practice. Participant A was active and when given mathematics tasks, he would quickly complete the task (with lots of mistakes) and started to talk with his friends who had not completed their work. The disruptive behaviour was contagious as other participants would lose concentration and joined him. Participant E liked to wander off the desk during lesson. Participant B and C were found daydreaming most of the time and were slow to respond.

The intervention activities managed to attract participants' attention. They were very engaged in the story about Chubby Bobby and Swingy Monkey with colourful and animated illustration. When they were posed questions related to the mathematical facts, they could associate and provide correct answers. They were not distracted during the whole session. In the following intervention sessions, they requested the story to be retold. They seemed to be fascinated by the characters and kept talking about the story in later sessions when doing the practice.

The hands-on activity successfully involved the participants like the Dienes Block used before the intervention. They were motivated to use the subtraction board to answer questions but a bit impatient when asked to use pictorial representation to substitute the concrete ice-cream sticks. They found the rap song funny but enjoyed singing it together. The most appealing activity was the subtraction game challenge in the computer. They collaborated and helped one another to score in the game.

In short, the change of attitude and learning behaviours in the classroom was apparent with the implementation of the intervention. The participants became more interested and confident in the learning.

Reflection and Discussion

As evident from the descriptive statistics from the pre- and posttest results as well as the findings from the follow-up interviews, the two-week intervention was successful in delivering the subtraction with regrouping concept and skills to the participants. Furthermore, it boosted their interest in learning.

The findings demonstrated that well-designed intervention can be beneficial to remedial pupils who require extra support in their learning. This result was consistent with Voza's (2011) claim that to build on memory retention, mathematical concepts should be introduced in context. In the intervention, contextual learning was applied in an interesting computer-illustrated story to attract participants' attention. The story could retain their memory about place values and the procedures involved in subtraction with regrouping. Using questioning techniques in the storytelling session, the concepts were successfully linked. Besides, the visual and audio elements helped to engage participants.

The CPA subtraction board further enhanced participants' earlier concept introduced in the story through hands-on manipulative activities, followed by visual representation in pictures and abstract computing in the final phase. The use of concrete manipulatives and pictorial representations made explicit the regrouping concepts and operations. Nevertheless, the scaffold should fade away once the participants grasp the concepts as the dependence on manipulatives and pictorial representations decelerated the computation. It was observed that some participants did not acquire fluent retrieval and took a longer time to solve a problem.

The song and rap predictably contributed to memory enhancement and served as a reminder of the sequential step in the computation. This was much more effective than teachers' repeated oral reminders, especially when the rap and singing was done in a humorous way with action. The lyrics would linger in the mind for longer period of time.

Another supporting element for the success was the computer game for skill rehearsing and practice. The drilling and practice became less tedious and more challenging when it was designed in a non-threatening game-like manner. This was in line with the brain-compatible learning principle (Caine, Caine, McClintic & Klimek, 2009). The incorporation of technology successfully boosted participants' learning interest and indirectly contributed to computation speed training for automaticity.

Conclusions and Recommendation

In conclusion, the action research proves that contextual learning could set the stage towards understanding of mathematical concepts while CPA gets the pupils systematically and progressively familiarize with the computation procedures. Scaffolds of manipulatives and drawings which are slowly removed could help pupils master the skills. Catchy song and challenging computer games serve as memory enhancer and intriguing drills for computation fluency and automatic retrieval. Putting all together, these variety of strategies successfully support pupils' knowledge, skills and motivation in learning Mathematics.

From the observation, some of the participants used ineffective counting which slowed down the computation. Future research could focus on explicit teaching of efficient counting strategies to achieve automaticity and fluent retrieval of mathematics fact.

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